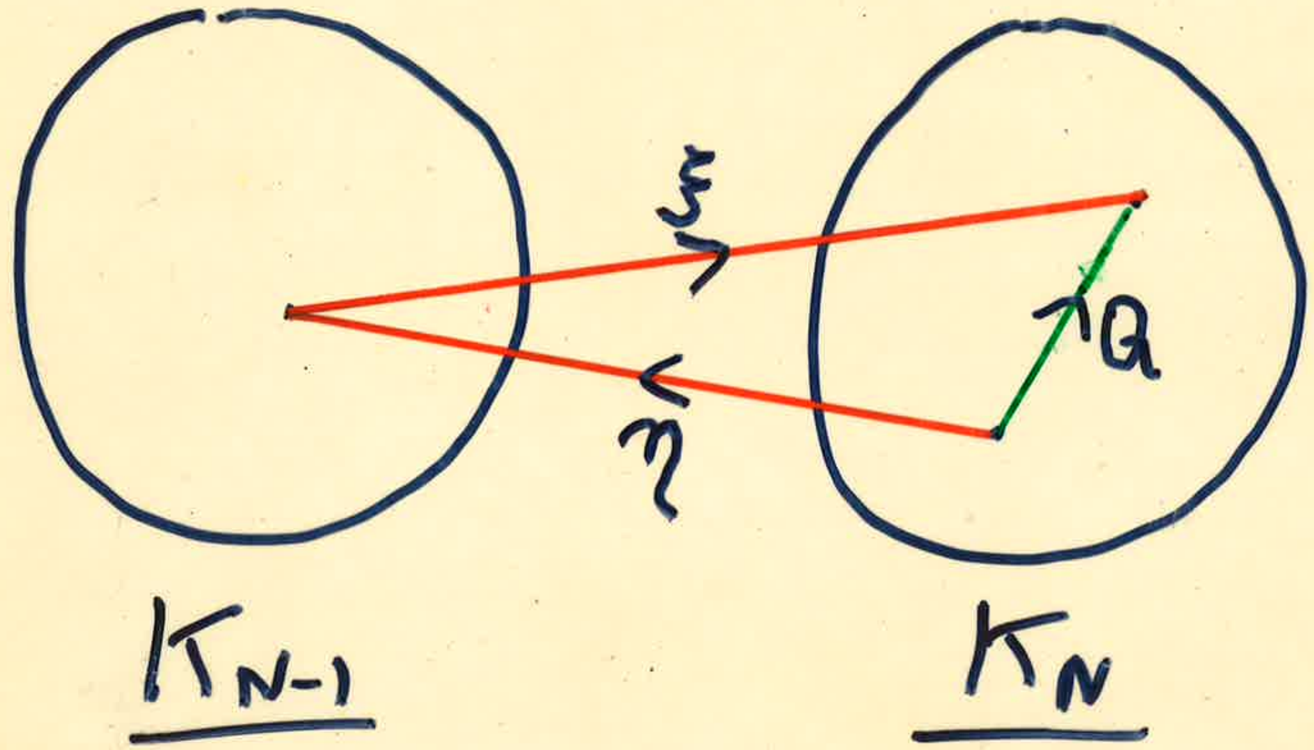
Schematically we factorise

$$Q = \xi \cdot \eta$$



But $Q' = \xi + \eta$ would create and annihilate particles

FIELD QUANTIZATION

Real Klein - Gordon field

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \mu^2\right)\psi = 0$$

Energy spectrum

$$E = \sum_{\underset{\sim}{k}} n_k (\hbar \omega_k) + \text{const.}$$

where $\text{const.} = \frac{1}{2}\sum_{\underset{\sim}{k}} (\hbar \omega_k)$

and $\omega_k = c\sqrt{\mu^2 + \underset{\sim}{k}^2}$

n_k are integral eigenvalues of the operator $N_k = a_k^{+} a_k$

where $a_k \,|n_k\rangle = \sqrt{n_k}\,|n_k - 1\rangle$

and $a_k^{+} \,|n_k\rangle = \sqrt{n_{k}+1}\,|n_k + 1\rangle$

<u>So</u> Basic Result of QFT is :

The Number of particles (quanta) present with momentum $(\hbar \underline{k})$ and energy $(\hbar \omega_{\underline{k}})$ in the <u>Particle Repnentation</u> is just the <u>excitation number</u> $n_{\underline{k}}$ of the $\underline{k}$ - mode

THE VACUUM

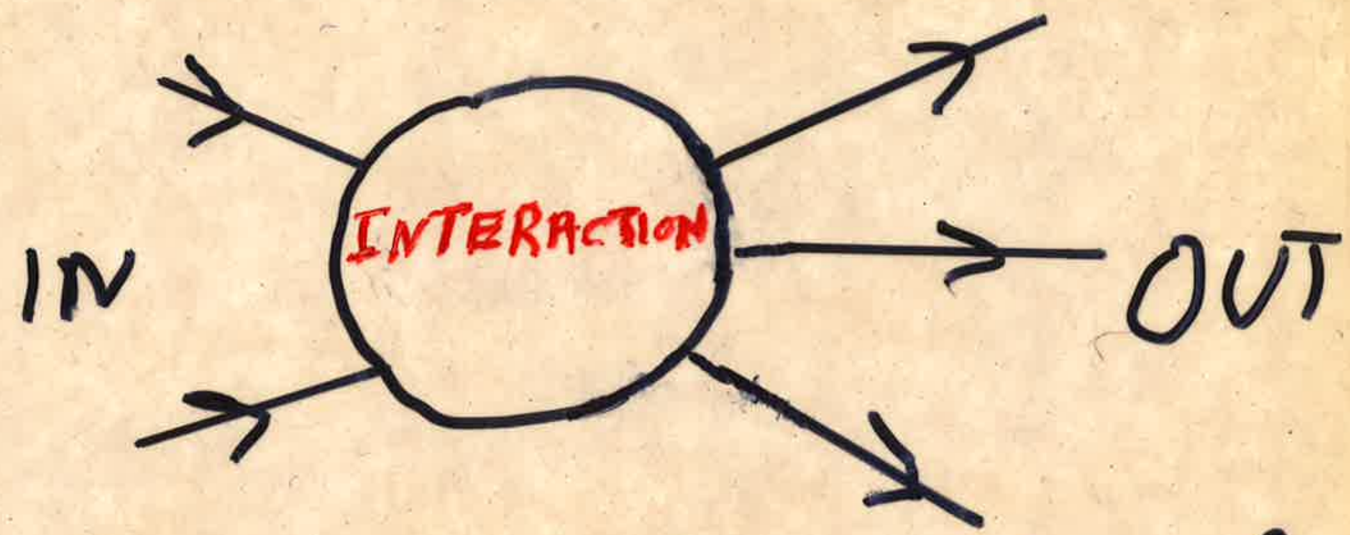
This is the state for which all the $n_{\underline{k}}$ are zero.

It is the lowest energy state of the field.

But the energy is not zero, since the field amplitude and other local quantities exhibit vacuum fluctuations.

The non-vanishing energy of the vacuum is called the zero-point energy of the field.

SCATTERING THEORY



The transition amplitude from the initial IN state to a final OUT state defines the S-Matrix

$$\langle OUT \mid S \mid IN \rangle$$

If $|\psi(t)\rangle = \sum_n c_n(t)\,|\phi_n\rangle$

where $|\phi_n\rangle$ are eigenstates of H_0, and

$$|\psi(-\infty)\rangle = |IN\rangle$$

Then the transition amplitude to the OUT state $|\phi_n\rangle$ is given by $c_n(\infty)$.

FEYNMAN DIAGRAMS

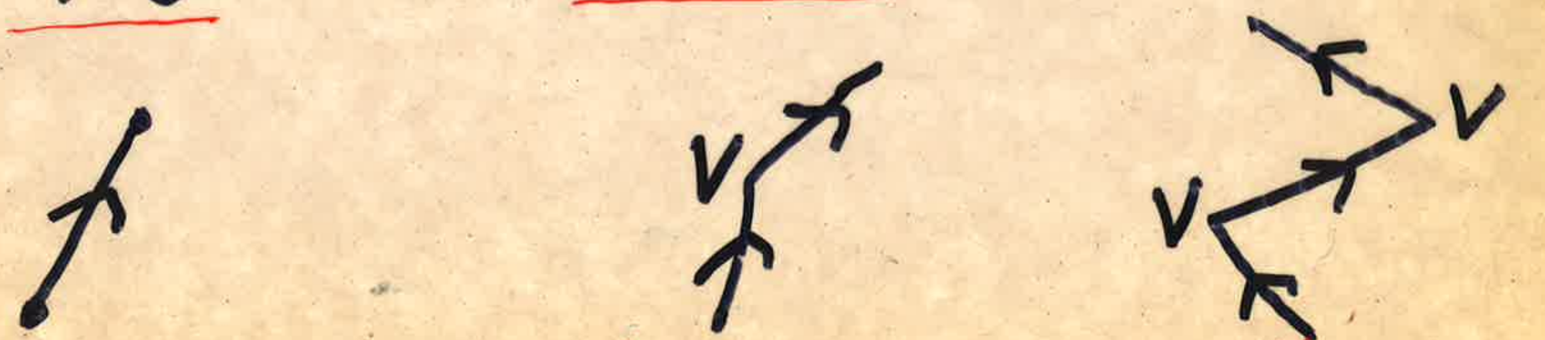
$$K = K_0 + K_0 V K$$

or $$(1 - K_0 V) K = K_0$$

So $$K = (1 - K_0 V)^{-1} K_0$$

$$= \sum_{n=0}^{\infty} (K_0 V)^n . K_0$$

$$= \underline{K_0} + \underline{K_0 V K_0} + \underline{K_0 V K_0 V K_0 \cdots}$$



No scattering

Single scattering

Double scattering

$$\langle 2 | K_0 | 1 \rangle \underset{\text{Df}}{=} K_0(2,1)$$

$$= \sum_n \phi_n^*(x_1)\, \phi_n(x_2)\, e^{-i/\hbar\, E_n (t_2 - t_1)} \cdot \theta(t_2 - t_1)$$

where $\theta(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$

$K_0(2,1)$ now satisfies the inhomogeneous equation $\left(i \frac{\partial}{\partial t_2} - H(x_2) \right) K_0(2,1) = \delta(t_2 - t_1) \cdot \delta(x_2 - x_1)$

so it is a genuine Green's function.

SECOND QUANTIZATION

Start with N-particle Wave Eq. for an assembly of Bosons.

State is specified by giving $n^{\underline{o}}$ n_i of particles in $i^{\underline{th}}$ 1-particle state $|u_i\rangle$ (with energy E_i)

Then $E = \sum_i n_i E_i$

Compare with assembly of harmonic oscillators

$$* \quad E = \sum_i \left(n_i + \tfrac{1}{2}\right) E_i \quad , \text{ if } \omega_i = E_i/\hbar$$

But * is what we would get by subjecting the 1-particle W. Eq. (such as K.G. Eq.) to a second quantization

THE TWO ROUTES TO QFT

'Real' field

Field Quantization

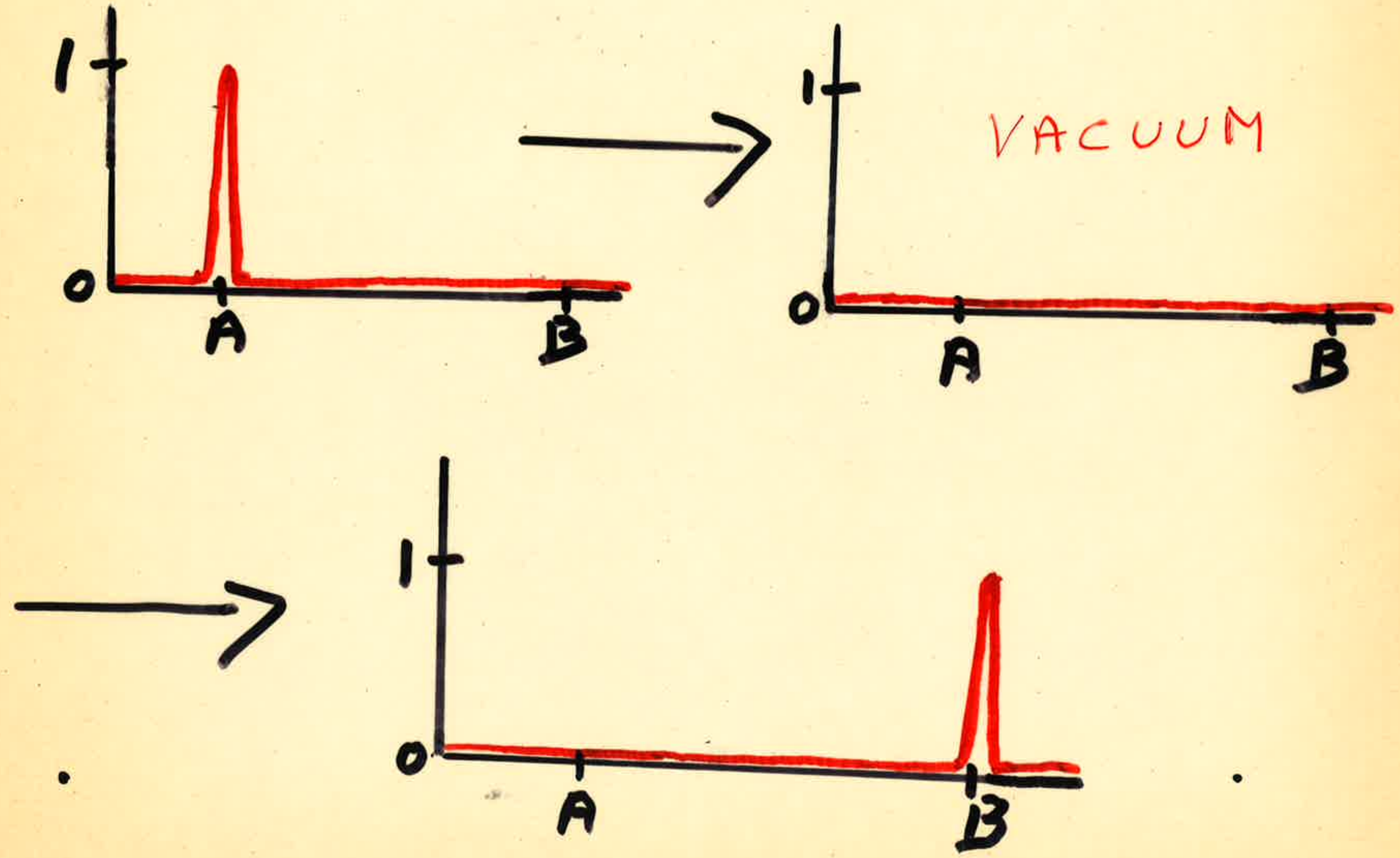
Quantum Field

N-Particle Schrödinger Eq.

2nd Quantization

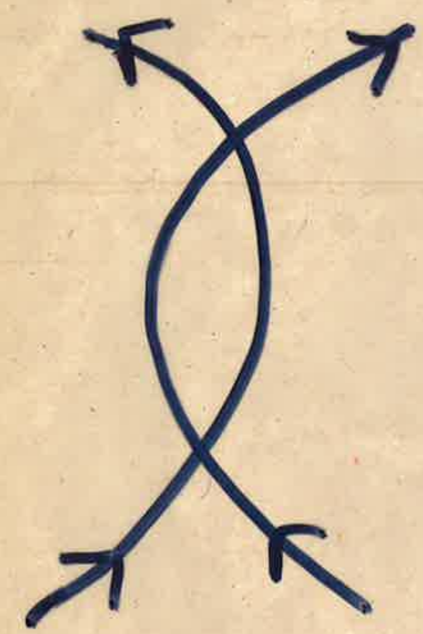
Query: Is Quantum Field the same animal in the two cases?

ANNIHILATION CREATION AND THE VACUUM

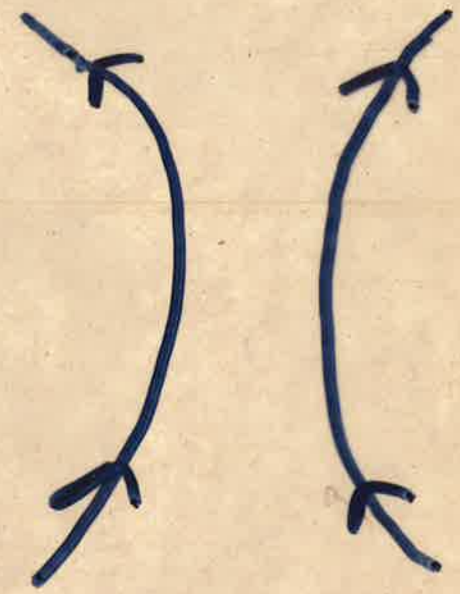


(17)

Does Exchange of Virtual Particles always produce Repulsion?



Attraction



Repulsion